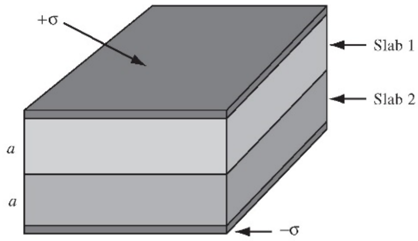
2018 Fall PHYS2310 電磁學（Electromagnetism) Midterm (double sides)

[Griffiths Chs. 4-7.1] 2019/01/10, 10:10am – 12:00am, 教師：張存續

- $\dfrac{1}{\mathscr{r}}=\dfrac{1}{R}\sum_{n=0}^{\infty}(\dfrac{r}{R})^n P_n(\cos\vartheta) \qquad r\le R$
- $\int_{-1}^{1} P_\ell(x)P_{\ell'}(x)dx=\int_0^{\pi} P_\ell(\cos\theta)P_{\ell'}(\cos\theta)\sin\theta d\theta=\begin{cases} 0 & \text{if } \ell'\neq\ell \\ \dfrac{2}{2\ell+1}, & \text{if } \ell'=\ell \end{cases}$

1. The space between the planes of a parallel-plate capacitor is filled with two slabs of linear dielectric material. Each slab has thickness a, so the total distance between the plates is $2a$. Slab 1 has a dielectric constant of 9, and slab 2 has a dielectric constant of 4. The free charge density on the top plate is σ and on the bottom plate $-\sigma$.



   (a) Find the electric displacement **D** in each slab. (5%)

   (b) Find the polarization **P** in each slab. (5%)

   (c) Find the potential difference between the metal plates. (5%)

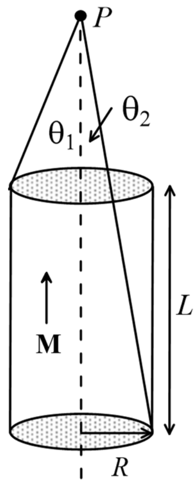
   (d) Find the location and amount of all bound charges (ρ_b and σ_b). (5%)

2. A sphere of radius R carries a polarization $\mathbf{P}(\mathbf{r})=P\hat{\mathbf{z}}$, where P is a constant and $\hat{\mathbf{z}}$ is the unit vector.

   (a) Calculate the bound charges ρ_b and σ_b .(10%)

   (b) Find the electric potential and field inside the sphere. (10%)

   [Hint: $V=\dfrac{1}{4\pi\varepsilon_0}\oint_S \dfrac{\sigma_b}{\mathscr{r}}\mathrm{d}a'+\dfrac{1}{4\pi\varepsilon_0}\int_v \dfrac{\rho_b}{\mathscr{r}}\mathrm{d}\tau'$]

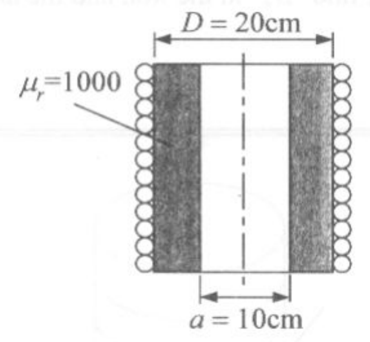


3. A bar magnet of radius R and length L is magnetized with a uniform magnetization **M** in the z axis as shown in the figure.

   (a) Find the bound volume current $\mathbf{J}_b$ inside the magnet as well as the bound surface currents $\mathbf{K}_b$ on both ends and the cylindrical surface. (10%)

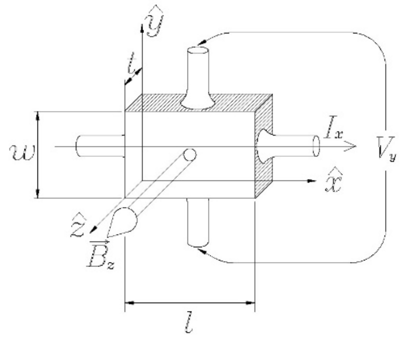
   (b) Find the magnetic field along the z axis. Use the technique similar to that of the solenoid and express you answer in terms of θ_1 and θ_2. (10%)

4. An *infinite* long magnetic material tube ($\mu_r = 1000$) of inner diameter a =10 cm and outer diameter D =20 cm is tightly wrapped with thin solenoid of 30 turns per unit cm, as shown in the figure. The current per turn I is 1 A. [Hint: $\mu_0 = 4\pi \times 10^{-7}$ H/m]



   (a) Find the auxiliary field **H** at the following three regions $0 \le r \le a/2$, $a/2 \le r \le D/2$, and $r \ge D/2$. (7%)
   (b) Find the magnetic field **B** at the following three regions $0 \le r \le a/2$, $a/2 \le r \le D/2$, and $r \ge D/2$. (7%)
   (c) Explain why the magnetic field **B** is discontinuous at the boundary $r = a/2$. (6%) [Hint: Use the boundary condition for **B** field.]

5. Consider a conducting slab as shown below with length l in the x direction, width w in the y direction and thickness t in the z direction. The conductor has charge carrier of charge q and charge carrier drift velocity v_x when a current I_x flows in the positive x direction. The conductor is placed in a magnetic field perpendicular to the plane of the slab $\mathbf{B} = B_z\hat{\mathbf{z}}$.



   (a) When steady state is reached, there will be no net flow of charge in the y direction. Find the relation between E_y, B_z, and v_x. (7%)
   (b) Find the resulting potential difference V_y (the **Hall voltage**) between the top and bottom of the slab, in terms of B_z, v_x, and the relevant dimensions of the slab. (7%)
   (c) How do you determine the sign of the mobile charge carriers in a material? (6%) [Hint: n denotes the number of carriers per unit volume]

6. Boundary Conditions
   (a) Write down the normal boundary condition $E^{\perp}$ and the tangential boundary conditions $\mathbf{E}^{||}$. [Hint: $\nabla \cdot \mathbf{E} = \rho/\varepsilon_0$ and $\nabla \times \mathbf{E} = 0$] (5%)
   (b) Write down the normal boundary condition $D^{\perp}$ and the tangential boundary conditions $\mathbf{D}^{||}$. [Hint: $\nabla \cdot \mathbf{D} = \rho_f$ and $\nabla \times \mathbf{D} = \nabla \times \mathbf{P}$] (5%)
   (c) Write down the normal boundary condition $B^{\perp}$ and the tangential boundary conditions $\mathbf{B}^{||}$. [Hint: $\nabla \cdot \mathbf{B} = 0$ and $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$] (5%)
   (d) Write down the normal boundary condition $H^{\perp}$ and the tangential boundary conditions $\mathbf{H}^{||}$. [Hint: $\nabla \cdot \mathbf{H} = -\nabla \cdot \mathbf{M}$ and $\nabla \times \mathbf{H} = \mathbf{J}_f$] (5%)